

Higgs Inflation in $f(\Phi, R)$ Theory

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Abstract

We have generalized the curvature coupling models of Higgs inflation to study inflation with a scalar field coupling of the form $\frac{\xi \Phi^a R^b}{M_p^{a+2b-4}}$. Such higher order scalar and graviton terms can be expected from the renormalization close to Planck scale. We compute the amplitude and spectral index of curvature perturbations generated during inflation and fix the parameters of the model by comparing these with the Planck+WP data. We find that if the scalar self coupling λ is in the range $(10^{-5} - 0.1)$, parameter a in the range $(2.3 - 3.6)$ and b in the range $(0.68 - 0.22)$ at the Planck scale, one can have a viable inflation model even for $\xi \simeq 1$. The tensor to scalar ratio r in this model is small and our model with scalar-curvature couplings is not ruled out by observational limits on r unlike the pure $\frac{\lambda}{4}\Phi^4$ theory. By requiring the curvature coupling parameter to be of order unity, we have evaded the problem of unitarity violation in scalar-graviton scatterings which plague the $\xi\Phi^2 R$ Higgs inflation models. We conclude that the Higgs field may still be a good candidate for being the inflaton in the early universe if one considers higher dimensional curvature coupling.

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1. INTRODUCTION

The idea that the universe through a period of exponential expansion, called inflation [1–6] has proved useful for solving the horizon and flatness problems of standard cosmology and in addition providing an explanation for the scale invariant super-horizon perturbations which are responsible of generating the CMB anisotropies and formation of structures in the universe. A successful theory of inflation requires a flat potential where a scalar field acquires a slow-roll over a sufficiently long period to enable the universe to expand by at least 60 e-foldings during the period of inflation. There is a wide variety of particle physics models which can provide the slow roll scalar field ‘inflaton’ for inflation [7]. From the observations of CMB anisotropy spectrum by COBE and WMAP [8] it is not yet possible to pin down a specific particle physics model as the one responsible for inflation. In the light of recent discoveries by CMS [9] and ATLAS [10] it is of interest to consider the Standard Model Higgs boson as the candidate for inflaton. On the face of it the idea does not work as the inflaton quartic coupling should be of the order $\lambda \sim 10^{-12}$ to explain the amplitude of CMB perturbations measured by WMAP [8] while the 125 GeV Higgs has a quartic coupling $\lambda \sim 0.13$ at the electroweak scale which can however go down to smaller values at the Planck scale due to renormalization [11–16]. However just from the standard model renormalization one cannot have the Higgs coupling $\lambda \sim 10^{-12}$ over the entire range of the rolling field $(10^{-1})M_P$ during inflation and the standard slow roll inflation with a Higgs field does not give the observed amplitude and spectrum of density perturbations [17]. If the Higgs and top mass are fine tuned then there can be a small kink in the Higgs potential and the universe trapped in this false vacuum can undergo a period of inflation [18–20].

A way out of fine tuning the scalar self coupling to unnaturally small values was given by Fakir and Unruh [21] who pointed out that if one couples the scalar field to the Ricci scalar $\xi\Phi^2R$ then the effective potential in the Einstein frame becomes a slow roll one with the effective scalar coupling being λ/ξ^2 . Density perturbations from inflation in the curvature coupled theories were calculated in [22, 23]. The equivalence of the density perturbation in Jordan and Einstein frame was shown by Komatsu and Futamase [24] who also calculated the tensor perturbations and showed that the tensor to scalar ratio is generically small in the Fakir-Unruh [21] model.

Bezrukov and Shaposhnikov [25–27] revived the large curvature coupling model to motivate the idea that the standard model Higgs field could serve as the inflaton in the early universe. The amplitude and spectral index of density perturbations observed by WMAP can be generated by the Higgs field with self coupling $\lambda \sim 0.1$ and curvature coupling $\xi \sim 10^4$ [25–28]. This large value of ξ needed however is seen as a problem as at the time of inflation the Higgs field is at the Planck scale and graviton-scalar scatterings due to the curvature coupling of the scalar would become non-unitary [29]. Ways of solving the unitarity violation problem in the Higgs inflation models have been explored in [30–42]. Higgs inflation with large scalar couplings have also been studied in [43, 44].

In this paper we assume that the dominant interaction between Higgs field and gravity is through operators of the form

$$\mathcal{L} = \frac{\xi(\mathcal{H}^\dagger\mathcal{H})^{a/2}R^b}{M_p^{a+2b-2}}. \quad (1)$$

The complete dynamics of the Higgs field involves the role of the Goldstone modes as has been studied in detail in [45–47]. The multifield dynamics of the Goldstone modes gives rise to sizable non-gaussianity. We will study the dynamics of the Higgs mode and impose a charge conservation and CP symmetry such that the Goldstone modes of the Higgs field do not acquire vevs. We will take the background Higgs field to be

$$\mathcal{H} = \begin{pmatrix} 0 \\ \Phi \end{pmatrix} \quad (2)$$

where Φ is the Higgs mode with mass 126 GeV. Our inflation model falls in the class of inflation in $f(\Phi, R)$ theories studied in Ref. [48]. Our motivation is that we use the Higgs quartic coupling $\lambda(\mathcal{H}^\dagger \mathcal{H})^2$ where the standard model value of $\lambda(\mu \sim M_P)$ can lie in the range $\lambda = (10^{-5} - 0.1)$ depending on the value of top quark mass [15, 16] or on new physics [49]. We take curvature coupling ξ to be of order unity and check the possibility of generating the observed density perturbations from Higgs inflation by varying parameters a and b .

We derive the curvature perturbation during inflation in two different ways. We derive the perturbations of modified Einsteins field equation in the Jordan frame in presence of the Higgs-curvature interaction terms and derive the amplitude and spectral index of curvature perturbation. We find that to generate the Planck+WP preferred amplitude $\Delta_{\mathcal{R}}^2 = 2.1955_{-0.585}^{+0.533} \times 10^{-9}$ and spectral index $n_s = .9603 \pm .0073$ [50] for $\lambda = 10^{-3}$ we should have $a \sim 3.02, b \sim 0.49$ (and for $\lambda = 0.1$ we need $a \sim 3.56, b \sim .22$). In these fits we take $\xi = 1$. Next as a consistency check we make a conformal transformation which removes the higher dimensional Higgs-curvature term but the effect of this term appears in the potential in the Einstein frame. We calculate the curvature perturbation in the Einstein frame and find that our results are consistent with the Jordan frame results. Calculation of the curvature perturbation in both Einstein and Jordan frame for the $\xi\Phi^2 R$ theory has been done previously in Ref. [51–54]. In the section (2) we derive the curvature perturbations and tensor perturbation in our theory in the Jordan frame and in the section (3) we make a conformal transformation to go to the Einstein frame and compute the curvature perturbations. Further in section (4) we discuss the renormalization of $\xi\Phi^2 R$ theory. Finally in the last section (5) we compare the results of the two different frames and discuss the viability of the Higgs inflation model in presence of higher order gravitational couplings.

2. CALCULATION IN THE JORDAN FRAME

In this section we introduce a scalar-gravity interaction term $f(\Phi, R)$ in the action and calculate physical quantities related to the inflationary density perturbations such as the spectral index, curvature perturbation and tensor-to-scalar ratio. We start with the action for a scalar field interacting with gravity of the form

$$S = \int d^4x \sqrt{-g} \left[-\frac{f(\Phi, R)}{2\kappa^2} + \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V(\Phi) \right], \quad (3)$$

where we take,

$$f(\Phi, R) = R + \frac{\xi \Phi^a R^b}{M_p^{a+2b-2}} \quad ; \quad V(\Phi) = \frac{\lambda \Phi^4}{4}, \quad (4)$$

where ξ is a dimensionless coupling constant. Varying the action (3) w.r.t $g^{\mu\nu}$ and Φ we obtain the field equations,

$$G_{\mu\nu} = FR_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - \nabla_\mu \nabla_\nu F + g_{\mu\nu} \square F = \kappa^2 \left(\nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2}g_{\mu\nu} \nabla^\rho \Phi \nabla_\rho \Phi - Vg_{\mu\nu} \right), \quad (5)$$

$$\square \Phi = V_{,\Phi} - \frac{f_{,\Phi}}{2\kappa^2}, \quad (6)$$

where $F = \partial f / \partial R = 1 + \frac{\xi b \Phi^a R^{b-1}}{M_p^{a+2b-2}}$.

2.1. Background quasi- De Sitter solution

For the unperturbed background FRW metric $(-1, a^2(t), a^2(t), a^2(t))$ and scalar field $\Phi = \phi(t)$, the above Eqs. (5) and (6) reduce to the form

$$3FH^2 + \frac{1}{2}(f - RF) + 3H\dot{F} = \kappa^2 \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right) \quad (7)$$

$$-2F\dot{H} - \ddot{F} + H\dot{F} = \kappa^2 \dot{\phi}^2 \quad (8)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} - \frac{f_{,\phi}}{2\kappa^2} = 0. \quad (9)$$

Now we assume the second term of F i.e. $\frac{\xi b \phi^a R^{b-1}}{M_p^{a+2b-2}}$ is dominant for some values of a and b . We find this assumption to be valid while solving numerically for the values of a and b in our model which give rise to the experimentally observed density perturbations as shown in the Table (I). From Eq. (7), under this assumption and considering the slow roll parameters which are defined in Eq. (28) as small, the Hubble parameter in the Jordan frame turns out to be of the form

$$H = \frac{\lambda^{\frac{1}{2b}}}{\sqrt{12}[\xi(2-b)]^{\frac{1}{2b}}} \left(\frac{\phi}{M_p} \right)^{\frac{4-a}{2b}} M_p. \quad (10)$$

From Eq. (9) under the slow roll assumption we get

$$\dot{\phi} = -\frac{\lambda \phi^3}{3H} \left[1 - \frac{a}{2(2-b)} \right]. \quad (11)$$

2.2. Scalar field and metric perturbations

Now we perturb Eqs. (5) and (6) by perturbing the scalar field $\Phi = \phi(t) + \delta\phi(x, t)$ and the metric as

$$ds^2 = -(1 + 2\alpha)dt^2 - 2a(t)(\partial_i \beta)dt dx^i + a^2(t)(\delta_{ij}(1 + 2\psi) + 2\partial_i \partial_j \gamma) dx^i dx^j, \quad (12)$$

where, α , ψ , β and γ are scalar perturbations. We derive the Einstein equations for the $f(R, \phi)$ theory [55, 56] keeping the first order terms in the metric and scalar field perturbations. The component δG_{00} is given by

$$\frac{\Delta}{a^2(t)}\psi + HA = \frac{-1}{2F} \left[\left(3H^2 + 3\dot{H} + \frac{\Delta}{a^2(t)} \right) \delta F - 3H\delta\dot{F} + \frac{1}{2} (2\kappa^2 V_{,\phi} - f_{,\phi}) \delta\phi \right. \\ \left. + \kappa^2 \dot{\phi} \delta\dot{\phi} + (3H\dot{F} - \kappa^2 \dot{\phi}^2) \alpha + \dot{F} A \right], \quad (13)$$

and taking the difference $\delta G_i^i - \delta G_0^0$ we get

$$\dot{A} + 2HA + \left(3\dot{H} + \frac{\Delta}{a^2(t)} \right) \alpha = \frac{1}{2F} \left[3\delta\ddot{F} + 3H\delta\dot{F} - \left(6H^2 + \frac{\Delta}{a^2(t)} \right) \delta F + 4\kappa^2 \dot{\phi} \delta\dot{\phi} \right. \\ \left. + (-2\kappa^2 V_{,\phi} + f_{,\phi}) \delta\phi - 3\dot{F}\dot{\alpha} - \dot{F} A \right. \\ \left. - \left(4\kappa^2 \dot{\phi}^2 + 3H\dot{F} + 6\ddot{F} \right) \alpha \right] \quad (14)$$

where $A = 3(H\alpha - \dot{\psi}) - \Delta\chi/a^2(t)$ and $\chi = a(t)(\beta + a\dot{\gamma})$. Here, in arriving the Eqs. (13) and (14), the leading order Eqs. (7) and (8) are also used. The other components δG_{i0} and δG_{ij} ($i \neq j$) of the first order perturbed Einstein equation can be written as

$$H\alpha - \dot{\psi} = \frac{1}{2F} \left[\kappa^2 \dot{\phi} \delta\dot{\phi} + \delta\dot{F} - H\delta F - \dot{F}\alpha \right], \quad (15)$$

and

$$\dot{\chi} + H\chi - \alpha - \psi = \frac{1}{F} (\delta F - \dot{F}\chi) \quad (16)$$

respectively. The equation of motion of scalar perturbation is

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \left[-\frac{\Delta}{a^2(t)} + \left(\frac{2V_{,\phi} - f_{,\phi}/\kappa^2}{2} \right) \right] \delta\phi = \dot{\phi}\dot{\alpha} + (2\ddot{\phi} + 3H\dot{\phi}) \alpha + \dot{\phi}A + \frac{1}{2}F_{,\phi} \left(\frac{\delta R}{\kappa^2} \right). \quad (17)$$

where

$$\delta R = -2 \left[\dot{A} + 4AH + \left(\frac{\Delta}{a^2(t)} + 3\dot{H} \right) \alpha + 2\frac{\Delta}{a^2(t)}\psi \right] \quad (18)$$

Now we analyze the curvature perturbation $\mathcal{R} = \psi - H\delta\phi/\dot{\phi}$ by choosing a gauge where $\delta\phi = 0$ and $\delta R = 0$. This sets $\mathcal{R} = \psi$ and moreover we have $\delta F = 0$ via $\delta F = (\partial F/\partial\phi) \delta\phi + (\partial F/\partial R) \delta R$. Under this gauge the Eq. (15) gives,

$$\alpha = \frac{\dot{\mathcal{R}}}{H + \dot{F}/(2F)} \quad (19)$$

and hence from Eq. (13) we get

$$A = -\frac{1}{H + \dot{F}/(2F)} \left(\frac{\Delta}{a^2(t)} \mathcal{R} + \frac{(3H\dot{F} - \kappa^2\dot{\phi}^2) \dot{\mathcal{R}}}{2F (H + \dot{F}/(2F))} \right). \quad (20)$$

Using Eq. (8) and Eq. (14), we obtain

$$\dot{A} + \left(2H + \frac{\dot{F}}{2F} \right) A + \frac{3\dot{F}}{2F} \dot{\alpha} + \left(\frac{3\ddot{F} + 6H\dot{F} + \kappa^2\dot{\phi}^2}{2F} + \frac{\Delta}{a^2(t)} \right) \alpha = 0. \quad (21)$$

Now we may write the differential equation for curvature perturbation by using the above Eqs. (19), (20) and (21) as

$$\ddot{\mathcal{R}} + \frac{(a^3(t)Q_s) \dot{\mathcal{R}}}{a^3(t)Q_s} + \frac{k^2}{a^2(t)} \mathcal{R} = 0, \quad (22)$$

where,

$$Q_s = \frac{\dot{\phi}^2 + 3\dot{F}^2/(2\kappa^2 F)}{(H + \dot{F}/(2F))^2}. \quad (23)$$

In arriving Eq. (22), Eq. (8) is again used. Now one may re-write the Eq. (22) in terms of variables $\omega = a\sqrt{Q_s}$ and $\sigma_k = \omega\mathcal{R}$ as

$$\sigma_k'' + \left(k^2 - \frac{\omega''}{\omega} \right) \sigma_k = 0, \quad (24)$$

where prime denotes the derivative with respect to the conformal time defined as $d\eta = dt/a(t)$ and

$$\frac{\omega''}{\omega} = \frac{a''(t)}{a(t)} + \frac{a'(t)}{a(t)} \frac{Q_s'}{Q_s} + \frac{1}{2} \frac{Q_s''}{Q_s} - \frac{1}{4} \left(\frac{Q_s'}{Q_s} \right)^2 \quad (25)$$

under quasi de-Sitter expansion $a(\eta) = \frac{-1}{H\eta(1-\epsilon_1)}$ and hence $\frac{a''(t)}{a(t)} = \frac{1}{\eta^2} [2+3\epsilon_1]$ and $a'(t)/a(t) = a(t)H$. Therefore we have

$$\frac{\omega''}{\omega} = \frac{1}{\eta^2} \left[\nu_{\mathcal{R}}^2 - \frac{1}{4} \right] \quad (26)$$

where

$$\nu_{\mathcal{R}}^2 = \frac{9}{4} \left[1 + \frac{4}{3} (2\epsilon_1 + \epsilon_2 - \epsilon_3 + \epsilon_4) \right]. \quad (27)$$

In arriving at the above expression we have defined

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_3 = \frac{\dot{F}}{2HF}, \quad \epsilon_4 = \frac{\dot{E}}{2HE}; \quad (28)$$

$$E = F + \frac{3\dot{F}^2}{2\kappa^2\dot{\phi}^2} = \frac{Q_s(1 + \epsilon_3)^2}{\dot{\phi}^2/(FH^2)}. \quad (29)$$

Here ϵ_i are slow roll parameters and $\dot{\epsilon}_i$ terms have been neglected. Equation (24) then has solutions in the Hankel functions of order ν_R

$$\sigma = \frac{\sqrt{\pi|\eta|}}{2} e^{i(1+2\nu_R)\pi/4} [c_1 H_{\nu_R}^{(1)}(k|\eta|) + c_2 H_{\nu_R}^{(2)}(k|\eta|)] \quad (30)$$

Applying the Bunch-Davies boundary condition $\sigma(k\eta \rightarrow -\infty) = e^{ik\eta}/\sqrt{2k}$ we fix the integration constants $c_1 = 1$ and $c_2 = 0$. Using the relation $H_\nu(k|\eta|) = \frac{-i}{\pi} \Gamma(\nu) \left(\frac{k|\eta|}{2}\right)^{-\nu}$ for the super-horizon modes $k\eta \rightarrow 0$, we obtain the expression for the power spectrum for curvature perturbations is defined as

$$\mathcal{P}_{\mathcal{R}} = \frac{4\pi k^3}{(2\pi)^3} |\mathcal{R}|^2 \equiv \Delta_{\mathcal{R}}^2 \left(\frac{k}{a(t)H}\right)^{n_{\mathcal{R}}-1} \quad (31)$$

The amplitude of the curvature power spectrum turns out to be

$$\Delta_{\mathcal{R}} = \frac{1}{\sqrt{Q_s}} \left(\frac{H}{2\pi}\right) \quad (32)$$

and the spectral index is

$$n_{\mathcal{R}} - 1 = 3 - 2\nu_{\mathcal{R}} \simeq -4\epsilon_1 - 2\epsilon_2 + 2\epsilon_3 - 2\epsilon_4 \simeq -6\epsilon_1. \quad (33)$$

Using slow roll parameters, Eq. (23) can be simplified to the form $Q_s \simeq 6F\epsilon_3^2 M_p^2$ with $\frac{\kappa^2 \dot{\phi}^2}{FH^2} \ll 6\epsilon_3^2$ which will be justified later in the subsection (2.4). In our model of $f(\Phi, R)$ coupling we find that to the leading order we have the relation $\epsilon_1 \approx -\epsilon_3$ and $\epsilon_2 \approx -\epsilon_4$ and these relations are used in the calculation of perturbation amplitude and spectral index. Plugging the values H and $\dot{\phi}$ from Eqs. (10) and (11) into Eq. (28), the slow roll parameters can be written as

$$\epsilon_1 = b^{-1}(a-4)(2-b)^{(1-b)/b}(a+2b-4)\lambda^{(b-1)/b}\xi^{1/b} \left(\frac{\phi}{M_p}\right)^{\frac{a+2b-4}{b}} \quad (34)$$

$$\epsilon_2 = b^{-1}(2-b)^{(1-b)/b}(a+2b-4)(a+6b-4)\lambda^{(b-1)/b}\xi^{1/b} \left(\frac{\phi}{M_p}\right)^{\frac{a+2b-4}{b}}. \quad (35)$$

For our model, we can write the expressions for the amplitude of power spectrum and the number of e-folding as

$$\Delta_{\mathcal{R}}^2 = \frac{b[(2-b)/\lambda]^{3-\frac{4}{b}} M_p^{8+\frac{4(a-4)}{b}} \xi^{-\frac{4}{b}} \phi^{-\frac{4(a+2b-4)}{b}}}{288(a-4)^2(a+2b-4)^2\pi^2} \quad (36)$$

and

$$N_J = \int_{\phi_f}^{\phi_J} \frac{H}{\dot{\phi}} d\phi = \frac{b[(2-b)/\lambda]^{\frac{b-1}{b}} \xi^{-\frac{1}{b}}}{2(a+2b-4)^2} \left(\frac{\phi}{M_p}\right)^{\frac{4-a-2b}{b}} \bigg|_{\phi_f}^{\phi_J} \quad (37)$$

respectively. Here ϕ_J and ϕ_f are the values of scalar field ϕ at the beginning and the end of inflation respectively.

2.3. Tensor Perturbations

We define the perturbation of metric as follows

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad \text{and} \quad g^{\mu\nu} = \bar{g}^{\mu\nu} + h^{\mu\nu}, \quad (38)$$

where $\bar{g}_{\mu\nu}$ is background metric and

$$h^{ij} = -\frac{1}{a^4(t)}h_{ij}, \quad h^{i0} = \frac{1}{a^2(t)}h_{i0}, \quad h^{00} = -h_{00}. \quad (39)$$

Now to get the equation of tensor perturbation, we set $h_{i0} = h_{00} = 0$ in the calculation. From the decomposition theorem, the non zero spatial components h_{ij} are traceless and divergenceless, i.e.,

$$h_{ii} = 0, \quad \partial_i h_{ij} = 0. \quad (40)$$

Using Eqs. (39) and (40), we obtain

$$\delta R_{00} = 0, \quad \delta R_{i0} = 0, \quad (41)$$

$$\delta R_{ij} = -\frac{1}{2a^2(t)}\nabla^2 h_{ij} + \frac{1}{2}\ddot{h}_{ij} - \frac{\dot{a}}{2a}\dot{h}_{ij} + 2\left(\frac{\dot{a}}{a}\right)^2 h_{ij}, \quad \delta R = 0. \quad (42)$$

So, perturbing Eq. (5), we obtain

$$\begin{aligned} \frac{1}{2}F a^2 \ddot{D}_{ij} + \left(\frac{1}{2}\dot{F}a^2 + \frac{3}{2}a\dot{a}F\right) \dot{D}_{ij} - \frac{F}{2}\nabla^2 D_{ij} = & \left[2\frac{\dot{a}}{a}\dot{F} - 2F\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}F + \frac{f}{2}\right. \\ & \left. + \ddot{F} + \frac{\kappa^2}{2}(\dot{\phi}^2 - 2V)\right] a^2 D_{ij}, \end{aligned} \quad (43)$$

where $D_{ij} = h_{ij}/a^2$. The right hand side of Eq. (43) vanishes by Eqs. (7) and (8). Thus we have

$$\ddot{D}_{ij} + \frac{(a^3 F)^\cdot}{a^3 F} \dot{D}_{ij} + \frac{\kappa^2}{a^2} D_{ij} = 0. \quad (44)$$

In the terms of polarization tensors e_{ij}^1 and e_{ij}^2 , the tensor D_{ij} is written as

$$D_{ij} = D_1 e_{ij}^1 + D_2 e_{ij}^2 \quad (45)$$

For gravity wave propagating in \hat{z} direction, the components of polarization tensor are given by

$$e_{xx}^1 = -e_{yy}^1 = 1, \quad e_{xy}^2 = e_{yx}^2 = 1, \quad e_{iz}^{1,2} = e_{zi}^{1,2} = 0. \quad (46)$$

So the Eq. (44) can be written as

$$\ddot{D}_\lambda + \frac{(a^3 F)^\cdot}{a^3 F} \dot{D}_\lambda + \frac{\kappa^2}{a^2} D_\lambda = 0. \quad (47)$$

where $\lambda \equiv 1, 2$. Now substituting $z = a\sqrt{F}$ and $v_k = zD_\lambda M_P/\sqrt{2}$, we get

$$v_\lambda'' + \left(k^2 - \frac{z''}{z}\right) v_\lambda = 0, \quad (48)$$

where, prime ' is derivative with respect to conformal time. Summing over all polarization states, the Eq. (48) provides us the amplitude of power spectrum of D_λ as

$$P_T = 4 \times \left(\frac{2}{M_p^2}\right) \frac{\kappa^3}{2\pi^2} \frac{1}{a^2 F} v_\lambda^2 \simeq \frac{2}{\pi^2} \left(\frac{H}{M_P}\right)^2 \frac{1}{F}. \quad (49)$$

So, the ratio of the amplitude of tensor perturbations to scalar perturbations r for $f(\Phi, R)$ theories is given by

$$r \simeq \frac{8\kappa^2 Q_s}{F} \simeq 48\epsilon_3^2. \quad (50)$$

We now use the measured values of these CMB anisotropy parameters to get the numerical values for the parameters (a, b, ξ, λ) .

2.4. Comparison with data

The recent Planck+WP data [50] gives the curvature perturbation $\Delta_{\mathcal{R}}^2 = 2.195_{-0.585}^{+0.533} \times 10^{-9}$, spectral index $n_{\mathcal{R}} = 0.9603 \pm 0.0073$ and $r < 0.11(95\%CL)$. We calculate the amplitude of the perturbation of the modes which leave the horizon with $N_J \simeq 70$. To get 70 e-foldings, the scalar field should roll from $1.6M_p$ to $1M_p$. Using Eq. (33) and Eq.(36), we calculate numerically the values of a and b for different values of self coupling λ which give the Planck+WP central values of $\Delta_{\mathcal{R}}^2 = 2.195_{-0.585}^{+0.533} \times 10^{-9}$ and spectral index $n_{\mathcal{R}} = 0.9603 \pm 0.0073$. We display these correlated set of parameters in Table (I). The slow roll parameters are found to be $\epsilon_1 \simeq -\epsilon_3 \simeq 0.0066$ and $\epsilon_2 \simeq -\epsilon_4 \simeq -0.0133$ for each value of λ . The value for the tensor to scalar ratio is also independent of λ and turns out to be $r \simeq 0.002$.

The Hubble parameter H in the Jordan frame turns out to be in the range $\sim (10^{-5} - 10^{-4})M_p$ as λ varies in the range $(10^{-5} - 0.1)$. The values of F are found to be much larger than unity and hence our assumption of dropping the unity in the expression for F is justified. This sets the order of the term $\frac{\kappa^2 \dot{\phi}^2}{FH^2} \sim 10^{-12}$ much smaller than ϵ_3^2 .

We have checked that in the limit $a \simeq 2$ and $b \simeq 1$ the correct value of $\Delta_{\mathcal{R}}$ and $n_{\mathcal{R}}$ are obtained for $\lambda \sim 10^{-2}$ only for large values $\xi \sim 10^4$. Our Jordan frame calculation in this limit is consistent with the results of [25–28] who do the calculation in the Einstein frame.

We shall next compare the results of these parameters in the Einstein frame and discuss the implications of these values in the concluding section.

λ	0.1	10^{-2}	10^{-3}	10^{-4}	10^{-5}
a	3.56398962	3.27512993	3.02576840	2.80955860	2.62084430
b	0.21800513	0.36243482	0.48711506	0.5952182	0.6895696
F	17130	7286	2779	1008	355
H	$6.24 \times 10^{-4} M_p$	$4.07 \times 10^{-4} M_p$	$2.5 \times 10^{-4} M_p$	$1.5 \times 10^{-4} M_p$	$8.89 \times 10^{-5} M_p$

TABLE I: The values of parameters in the Jordan frame at $N_J(\phi_J) \simeq 70$ with $\xi = 1$ and $\phi_J = 1.6 M_p$ for different values of λ .

3. CALCULATION IN THE EINSTEIN FRAME

Starting with the considered action

$$S = \int d^4x \sqrt{-g} \left[-\frac{M_p^2}{2} R \left(1 + \frac{\xi \Phi^a R^{b-1}}{M_p^{a+2b-2}} \right) + \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{\lambda \Phi^4}{4} \right] \quad (51)$$

we perform a conformal transformation of the metric $g_{\mu\nu}$ to the Einstein frame metric $\tilde{g}_{\mu\nu}$ which is defined as

$$\tilde{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x) , \quad (52)$$

where

$$\Omega^2 = 1 + \frac{\xi \Phi^a R^{b-1}}{M_p^{a+2b-2}} . \quad (53)$$

The Ricci scalar transform as

$$R = \Omega^2 \left[\tilde{R} + 6 \frac{\tilde{\square} \Omega}{\Omega} - 12 \frac{\tilde{\partial}^\mu \Omega \tilde{\partial}_\mu \Omega}{\Omega^2} \right] . \quad (54)$$

For quasi de-Sitter space we can ignore the second and third terms in the bracket in Eq. (54) which is justified in the Eq. (67). For this slow roll case, we can write Eq. (53) in Einstein frame as

$$\Omega^2 = 1 + \frac{\xi^{1+\beta} \Phi^\alpha \tilde{R}^\beta}{M_p^{\alpha+2\beta}} , \quad (55)$$

where, $\alpha = a/(2-b)$ and $\beta = (b-1)/(2-b)$. Now we write the action (51) in term of new field χ , which is related to the field Φ by the relation

$$\frac{d\chi}{d\Phi} = \frac{1}{\Omega^2} \left(\Omega^2 + \frac{3\alpha^2 \xi'^2}{2} \left(\frac{\Phi}{M_p} \right)^{2\alpha-2} \right)^{1/2} , \quad (56)$$

where $\xi' = \xi^{1+\beta}(\tilde{R}/M_p^2)^\beta$. This leads the action in term of χ as follows

$$S_E = \int d^4x \left(-\frac{M_p^2}{2} \tilde{R} + \frac{1}{2} \tilde{D}^\mu \chi \tilde{D}_\mu \chi + U(\chi) \right), \quad (57)$$

where

$$U(\chi) = \frac{1}{\Omega^4} \frac{\lambda}{4} \Phi(\chi)^4. \quad (58)$$

For $\Phi \gg M_P/\xi'^{1/\alpha}$, Eq. (56) can be integrated to give

$$\Phi = \frac{M_p}{\xi'^{1/\alpha}} \exp \left(\sqrt{\frac{2}{3}} \frac{\chi}{M_p \alpha} \right). \quad (59)$$

Considering $\tilde{g}_{\mu\nu} = (-M^2(t), a^2(t), a^2(t), a^2(t))$ and varying the action (57) with respect to $M(t)$ or $a(t)$ and setting $M = 1$ in the final equation which corresponds FRW metric, we get the Friedmann equation

$$12\tilde{H}^2 - \zeta^{-1} M_p^2 \lambda \left(1 + \frac{2\beta}{\alpha} \right) = 0, \quad (60)$$

where

$$\zeta = 12^{4\beta/\alpha} \left(\frac{\tilde{H}^2}{M_p^2} \right)^{4\beta/\alpha} \xi^{\frac{4(1+\beta)}{\alpha}} \exp \left(2\sqrt{\frac{2}{3}} \frac{(\alpha-2)\chi}{\alpha M_p} \right). \quad (61)$$

Here we have neglected all the derivative terms of Hubble parameter \tilde{H} . This corresponds to slow roll condition, i.e., $\dot{\chi}^2$ is much smaller than potential term. We may write the Hubble parameter from Eq. (60) as

$$\tilde{H} = M_p \frac{[(1+2\beta/\alpha)\lambda]^{\frac{\alpha}{2(\alpha+4\beta)}}}{\sqrt{12} \xi^{\frac{2(1+\beta)}{\alpha+4\beta}}} \exp \left[\sqrt{\frac{2}{3}} \left(\frac{2-\alpha}{\alpha+4\beta} \right) \frac{\chi}{m} \right]. \quad (62)$$

Now using Eq. (62) and (58) we obtain

$$U(\chi) = \frac{1}{4} M_p^4 \lambda^{\frac{\alpha}{\alpha+4\beta}} \xi^{-\frac{4(1+\beta)}{\alpha+4\beta}} \left(1 + \frac{2\beta}{\alpha} \right)^{-\frac{4\beta}{\alpha+4\beta}} \exp \left[2\sqrt{\frac{2}{3}} \left(\frac{2-\alpha}{\alpha+4\beta} \right) \frac{\chi}{M_p} \right]. \quad (63)$$

Here we have taken the approximation $\exp(\sqrt{\frac{2}{3}} \frac{\chi}{M_p}) \gg 1$ for $\chi \gg M_p$. We now compute the spectral index and curvature perturbation using above potential (63). The slow roll parameters for large $\chi \gg M_p$ comes out to be

$$\epsilon = \frac{M_p^2}{2} \left(\frac{U'}{U} \right)^2 = \frac{4}{3} \left(\frac{a+2b-4}{a+4b-4} \right)^2; \quad \eta = M_p^2 \left(\frac{U''}{U} \right) = \frac{8}{3} \left(\frac{a+2b-4}{a+4b-4} \right)^2 \quad (64)$$

$\lambda = 0.1$	$\lambda = 10^{-2}$	$\lambda = 10^{-3}$	$\lambda = 10^{-4}$	$\lambda = 10^{-5}$
$a = 3.395$	$a = 3.040$	$a = 2.750$	$a = 2.512$	$a = 2.310$
$b = 0.2727$	$b = 0.4327$	$b = 0.5630$	$b = 0.6708$	$b = 0.7616$

TABLE II: The values of parameters in the Einstein frame at $N_E(\phi_E) \simeq 70$ with $\xi = 1$ for different values of λ .

and the curvature perturbation

$$\begin{aligned}\Delta_{\mathcal{R}} &= \frac{3H^3}{2\pi U'(\chi)} \\ &= \frac{1}{8\sqrt{2}\pi} \left(\frac{y+2}{2y-x+4} \right)^{\frac{x+2y+4}{2x}} \lambda^{\frac{2y-x+4}{2x}} \xi^{-\frac{2}{x}} \left(\frac{x}{y} \right) \exp \left(-\sqrt{\frac{2}{3}} \frac{y\chi}{xM_p} \right),\end{aligned}\quad (65)$$

where $x = a + 4b - 4$ and $y = a + 2b - 4$.

The number of e-folding is calculated as

$$\begin{aligned}N_E &= \int_{\chi_e}^{\chi_0} \frac{U(\chi)}{U'(\chi)} d\chi \\ &= -\frac{1}{2} \sqrt{\frac{3}{2}} \left(\frac{x}{y} \right) \left(\frac{\chi_0 - \chi_e}{M_p} \right)\end{aligned}\quad (66)$$

For $\chi_0 \sim 15M_p$ and $\chi_e \sim 1M_p$, the number of e-folding is found to be around 70. The slow roll parameters ϵ , η and Hubble parameter H are nearly independent with λ and are ~ 0.02 , ~ 0.04 and $\sim 5.9 \times 10^{-5} M_p$ respectively.

Now from Eqs. (53) and (59), we can calculate the order of terms like $\ddot{\Omega}/\Omega$ and $(\dot{\Omega}/\Omega)^2$ for $\phi \gg \frac{M_p}{\xi^{1/\alpha}}$. For $\lambda = 10^{-3}$ and $\xi = 1$,

$$\begin{aligned}\frac{\ddot{\Omega}}{\Omega} &\sim \frac{\alpha(\alpha-2)}{4} \left(\frac{\dot{\phi}}{\phi} \right)^2 = \frac{\alpha(\alpha-2)}{6} \left(\frac{U'(\chi)}{3\alpha M_p H} \right)^2 = -2.33 \times 10^{-12} M_p^2 \quad \text{and} \\ \left(\frac{\dot{\Omega}}{\Omega} \right)^2 &\sim \frac{\alpha^2}{4} \left(\frac{\dot{\phi}}{\phi} \right)^2 = 5.16 \times 10^{-11} M_p^2\end{aligned}\quad (67)$$

respectively, whereas the value of curvature scalar $\tilde{R} = 12H^2$ at the same values of parameter is $4.24 \times 10^{-8} M_p^2$. Here we have neglected $\ddot{\phi}$ as it is small. Thus our approximation made is consistent and may be checked for other values of a and b .

4. RENORMALIZATION

During inflation the Higgs field is in the Planck regime so the running of the dimensionless parameters λ and ξ due to renormalization from the low energies to the Planck scale must

be taken into account [57–64]. What we find in this paper is that in order to reproduce the observed density perturbations we need not tune ξ to any large values as in ref. [21–28] but what is needed is to induce large anomalous dimensions in the scalar and graviton fields [65] at the Planck scale. The renormalization group corrected effective action can be written in the general form as

$$S = \int d^4x \sqrt{-g} \left[- (G_g(t)^2 R) \left(\frac{M_p^2 + \xi G_\Phi(t)^2 \Phi^2}{2} \right) + \frac{1}{2} G_\Phi(t)^2 (\partial_\mu \Phi)^2 + \frac{\lambda(t) G_\Phi(t)^4 \Phi^4}{4} \right], \quad (68)$$

where $t = \ln \left(\frac{\Phi}{\mu} \right)$ with μ being the renormalization scale and the wave-function renormalization factor

$$G_\Phi(t) = \exp \left(- \int \frac{\gamma_\Phi(t)}{1 + \gamma_\Phi(t)} dt \right), \quad (69)$$

where $\gamma_\Phi(t)$ is the anomalous dimension of the scalar field.

A similar expression holds for the graviton wave-function renormalization G_g . The parameters a and b can formally be related to wave-function renormalizations G_Φ and G_g as

$$a = 2 + 2 \frac{\partial \ln G_\Phi}{\partial \ln \Phi}; \quad b = 1 + 2 \frac{\partial \ln G_g}{\partial \ln g_{\mu\nu}}. \quad (70)$$

In this paper we show that the correct values of a and b which generate a viable inflation must lie in the range $a = (2.3 - 3.6)$, $b = (0.68 - 0.22)$ and we believe that renormalization effects should lead to these values of anomalous dimension of the scalar and graviton fields.

5. DISCUSSION AND CONCLUSION

We have generalised the curvature coupling models of Higgs inflation to study inflation with a scalar field for a $\frac{\lambda}{4}\Phi^4$ potential and a curvature coupling of the form $\frac{\xi \Phi^a R^b}{M_p^{a+2b-4}}$. This curvature coupling can be obtained for example by starting with a tree level coupling term $\xi \Phi^2 R$ ($a \simeq 2, b \simeq 1$) if the wave-function renormalization of Φ and the graviton introduce large anomalous dimensions for these fields such that a is increased to lie in the range $(2.3 - 3.6)$ and b is reduced to the range $(0.68 - 0.22)$. In this case if the renormalized λ lies in the range $(10^{-5} - 0.1)$ at the Planck scale, one can have a viable inflation model even for $\xi \sim 1$.

It has been shown that the Higgs self coupling can go from $\lambda = 0.13$ at the electroweak scale for the 125 GeV Higgs to $\lambda = 10^{-5}$ at the Planck scale by tuning the top mass or by introducing extra interactions [15, 16, 49]. This leads us to conclude that the Higgs field may be still a good candidate for being the inflaton in the early universe if one considers higher order curvature coupling which may arise from renormalization at the Planck scale.

The tensor to scalar ratio r in this model is small and the $\frac{\lambda}{4}\Phi^4$ with scalar curvature couplings is not ruled out by observational limits on r unlike the pure $\frac{\lambda}{4}\Phi^4$ theory [8, 66].

We find that the results of Jordan frame and Einstein frame calculations are in agreement up to leading order in the slow roll parameters as expected from the approximation made in the course of conformal transformation (54,55) (and justified in the Eq. (67)). Finally, by requiring the curvature coupling parameter to be of order unity, we have evaded the problem of unitarity violation in scalar-graviton scatterings [29] which plagued the $\xi\Phi^2 R$ Higgs inflation models [25–28].

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